Ordering Operators:
Towards a Discrete, Strictly Finite and Quantized Interpretation of the Tensor Calculus

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The Ordering Operator Calculus (1982) provided a discrete, strictly finite foundation for differential geometry and the calculus. This abstract mathematical formulation was subsequently incorporated in Foundations of a Discrete Physics (1988), which applied it to a reformulation of relativistic quantum mechanics, vis-à-vis the combinatorial hierarchy and Prof. Noyes' Bit String Physics. The present paper provides a review of ordering operators, discusses principles for applying the ordering operator calculus to physics, and provides a high-level introduction to an OOC notational calculus – the tensor calculus – so that tensor equations can be interpreted as a special case of ordering operator equations.

Preface

I first met Prof. Noyes ("Pierre") in the spring of 1978 at a weekly seminar I co-sponsored with Dr. Hewitt D. Crane at Stanford Research Institute (now SRI International) and initiated by my friend and co-author Eddie Oshins. Although I was familiar with the work of Dr. E. W. "Ted" Bastin, it was there Pierre brought us up-to-date on the Combinatorial Hierarchy and I first introduced him to the beginnings of the ordering operator calculus. Subsequently, ANPA was founded at Pierre's instigation, and he invited me to attend the second annual meeting at King's College, Cambridge University, where I had the honor of meeting Ted Bastin, Prof. Clive Kilmister, Dr. John Amson, Dr. Fredrick Parker-Rhodes, and others. I began working with Pierre at SLAC under his sponsorship in the early 1980s. Over the years, Pierre has been friend, mentor, teacher, and inspiration. In all likelihood, had it not been for Pierre, I would have abandoned my work in physics altogether and might well have abandoned the ordering operator calculus.

Thank you Pierre, and Happy 90th Birthday!

I. Introduction

The ordering operator calculus (OOC) is a purely discrete and finite mathematics, and provides a covering theory for certain theories of continuum mathematics. Proper subsets of OOC are isomorphic to those theories up to their requirements for infinities, infinitesimals, and unbounded procedures, the OOC having constructs that remove those requirements. Previous publications on the subject established a general foundation for the ordering operator calculus. Ordering operators were defined and the formalisn shown to be powerful enough to provide support for the apparatus of topology and differential geometry (The Ordering Operator Calculus, 1982–1985). This was done by recasting and redefining the foundations of finite differences, incorporating insights from recursion theory and combinatorics. Subsequent publications such as Foundations for a Discrete Physics (“FDP”), (SLAC-Pub 4526, June 1989) added conceptual discussion of ordering operators and explored applications, especially to physics and, in particular, special relativity, relativistic quantum mechanics and quantum field theory.

With the exception of the original OOC paper which introduced the abstract (*i.e.*, unapplied) mathematical concepts, previous papers have addressed results derivable when particular classes of ordering operators are used. In most of those publications pertaining to physics, the discussion was restricted to a special class of ordering operators that would reproduce Prof. Noyes' Bit String Physics in which bit strings partitionable into label and address portions are generated. In FDP, the conception of the input or output of ordering operators as being labels with unspecified structural complexity was a convenience not intrinsic to ordering operators in the general case. Many results obtained in FDP were already obtained in the more general case.

In prior publications, a unified apparatus for symbolic computation has not been given. That omission is corrected with this paper. As a by-product, OOC obtains application to general relativity and that application is born quantized. Mathematical details will be addressed in future papers as time permits.

II. Conceptual Beginnings

Many systems encountered in both theory and practice are a priori discrete, finite, and intrinsically process oriented. By discrete, I mean that there is no intrinsic reason to import or assume the properties of the continuum[1]. By finite, I mean that there is no reason for assuming any infinities (completed infinities) or infinite extensibility (*e.g.*, infinite recursion). With Leibnitz, we consider these to be fictions leading to computational shortcuts. Furthermore, certain properties may depend on the cardinality of the system and its subsystems. By intrinsically process oriented I mean that there is a inherent notion of the system evolving recursively in that certain

[1]One might argue, however, that the OOC approach is consistent with an Archimedean continuum based on ratios (as contrasted with the Wierstrass version based on limits).

properties depend on the generation or on comparison of generations. Examples are physical systems that display characteristic numbers of the Fibonacci series, Combinatorial Hierarchy, and the like.

In representing such systems, it is important to be very reluctant to introduce any continuum properties in the mathematical description, and to do so only deliberately and with full cognizance of their effects. Preferably, these effects can be contained or bounded so that they do not pervade the representation and erode (*e.g.*, contradict) the finite, discrete, and process properties. The received foundations of mathematics and logic introduce continuum properties in many ways, both within the object language and the metalanguage[2].

III. Physical Motivations

A. Problems with the Computational Apparatus

There exists a seemingly irresolvable tension between mathematics and science that is never more apparent than it is in the foundations of physics, where it becomes a violent collision. Many physicists choose to ignore these problems, treating them as inconsequential artifacts. A science should be questioned when its mathematics must be circumvented by procedures that have no physical motivation except to avoid absurdities. Problems such as:

1. the inherent incompatibility of quantization and geometrodynamics while both quantum theory and geometrodynamics give correct empirical results,

and

2. the appearance of infinities that require ad-hoc renormalization, especially when that renormalization leads to astoundingly accurate predictions as in quantum field theory,

should not be dismissed. These incompatibilities do not exist in the "real world" – whatever that might be, it is necessarily a self-consistent entity.

Although renormalization procedures are motivated by vague physical requirements (*e.g.*, bounding momentum or dimensionality), the choice of renormalization procedure is really little more than trial and error (*i.e.*, picking one that yields correct results) with heuristic motivation. How baldly embarrassing all this would be if it were it not for the astounding accuracy that obtains. Of course, all this occurs simply because some of our mathematical entities (*e.g.*, operators and variables) are ill-defined. When the complexity of a physical process like particle interaction is

[2]Indeed, Abraham Robinson's non-standard analysis takes this infection to the extreme and makes it inherent in the mathematics. I disagree with Robinson's historical perspectives, including the notion that infinitesimals are intuitive – that perspective came into being only in modern times after decades of indoctrination and, I dare say, is not held by the mathematically unsophisticated.

recursively extensible without bound (because you don't know when the recursion should halt based on any physical understanding), then no finite quantities characterizing the process have any meaning. As is well-known, this fact raises its ugly head precisely because we can't normalize probabilities in the first place. To put it bluntly, we can wave the magic wand of renormalization in QED or QCD to get something that works, but we really don't know what we are talking about.

I'm not being critical of my physicist and mathematician friends. These are very serious, very difficult problems. Most practicing physicists just want to get on with the business at hand and assume that all this will eventually sort itself out – or that it is anomalous in some way.

I submit that these problems arise in the first place because we have the wrong mathematical model on several counts: the continuum. To be clear, my position is that continuum mathematics as currently practiced by physicists is an inadequate tool for modeling the foundations of physics. This position has been held by numerous physicists and mathematicians including Weyl, Gödel, and Wheeler. According to Wheeler, Weyl believed the continuum of the natural numbers an idealization and that the lesson of Gödels incompleteness theorems was that we commit a folly when we construct or believe in completed infinities (*i.e.*, infinity as number).

B. Conflicts Between Ontology and Epistemology

Bohr seems to have taken an epistemological view of physics, choosing to understand the task of physics as being to describe the information we can have about reality, and going so far as to deny any reality. By contrast, Einstein's ontological view of physics poses the task of phyisics as being to describe reality. Neither can be completely correct nor are they strictly contradictory.

On the one hand, we are surely trapped in informational theories and can never directly perceive some assumed objective reality. All we can demand is epistemological consistency. In this sense, Bohr was right. However, he went too far in claiming that the limits of a particular epistemological theory (*e.g.*, Copenhagen quantum mechanics) were necessarily inviolate or that that epistemological theory could be uniquely correct. In particular, an assertion that reality is inherently probabilistic is simply unprovable – about as helpful as asserting that an omnipotent god is responsible for everything.

On the other hand, Einstein's desire to answer the ontological question is clearly more in line with our intuitive understanding of the task of physics. However, simultaneously postulating a continuum, infinite reality and a complete, deterministic descriptive theory is contradictory, while denying epistemological limitations, is contradictory. In this sense, Einstein was wrong. At best all physics can do is identify that class of theories powerful enough to describe our (presumed objective) experience of reality and eliminate those theories that contradict that experience. That is, the task of physics (and, in general, of any experimental science) is to tell us what reality is not.

I see the primary task of physics as being, first and foremost, to provide a unified approach to our understanding of information[3]. Then and only then can we consider information as an explanation of the causal structures we call reality. What is needed is an abstract mathematical apparatus for constructing abstractions that (a) is rich enough to model physical properties and their measurement and (b) can do so without a priori or imported abstractions or limitations.

IV. Ordering Operators

A. Graph Representation

For pedagogical purposes, an ordering operator is a recursive generator of a particular directed acyclic graph (DAG) defined on an ensemble of nodes of cardinality N. The graph is not embedded in any space. We call this particular DAG the ordering operator's canonical DAG (CDAG). There are two elements to this notion: (1) the canonical directed graph and (2) its generation. It is convenient to think of a specific ordering operator as a dedicated purpose computer which contains its canonical CDAG in internal memory and instructions for recursively providing a walk of that CDAG.

As is well known, a directed graph G containing n nodes (a.k.a. vertices) can be represented by an $n{\times}n$ adjacency matrix A which shows which node of the graphs are connected to which other nodes. Generally, the a_{ij} entry is the number of connections (a.k.a. arcs or edges) from the i^{th} node to the j^{th} node. Every arc has an initial node n_i and a final node n_j. A binary adjacency matrix A or connection matrix restricts the number of arcs from the i^{th} node to the j^{th} node to Boolean values of either 0 or 1. Thus, a connection matrix satisfies the first part of an ordering operator by representing the reachability relation for the graph. If the entries for the i^{th} column of A are all zero, then we say the i^{th} node is an *initial node* of G. If the entries for the j^{th} row of A are all zero, then we say that the j^{th} node is a *terminal node* of G.

Define an adjacency sub-matrix A_k for the binary adjacency matrix A as an $n{\times}n$ matrix in which some of the arcs represented in A are disallowed; that is, the corresponding entry is 0 in A_k where in A it was a 1 (or more).

The generation of a directed graph has a dual representation, either as a totally ordered set of states of the graph or as a totally ordered set of transition matrices.

An ordering operator may be conveniently thought of as a particular walk of a pre-existing DAG, although this does not capture the ontological content of the OOC and can be misleading[4]. An adjacency matrix contains insufficient information

[3] I do not mean information theory.

[4] Care must be taken not to ignore the generative aspects of the computation, which are essential to understanding the process aspects of an OOC application.

to capture more than one walk of the CDAG. Walking a graph consists in visiting each node of the graph in such a way that every arc is traversed at least once, subject to the requirement that the initial node of the arc is either an initial node of G or is a final node in the previous step. Every walk is either a walk on a directed graph or may be understood as inducing a direction on each arc of an undirected graph.

A *state* can be defined as a matrix showing which nodes have been most recently visited, beginning with those nodes of the graph that have only outbound arcs (*i.e.*, have only 0s in corresponding column of the adjacency matrix) and ending on those nodes which of the graph that have only inbound arcs (have only 0s in the corresponding row of the adjacency matrix). For a walk of graph G, define a state vector at step S_m of the walk as the n-element vector for which the i^{th} element is zero unless the most recent step has followed at least one arc that terminates at the corresponding node. By convention, we will express state vectors as column vectors.

The initial state of G is represented by the state vector S_i in which the j^{th} element is 1 if the j^{th} node is an initial node of G and is 0 otherwise. The final state of G is represented by the state vector S_f in which the j^{th} element is 1 if the j^{th} node is a terminal node of G and is 0 otherwise. In general, the n^{th} state of the ordering operator is represented by an n-vector in which every node that has been visited has a value of 1 and every node that has not been visited has value 0.

A *transition matrix* is a projection of the adjacency matrix that delineates a single step in generating the adjacency matrix. Combining the set of transition matrices in order yields the adjacency matrix. In order to capture the evolution of the DAG, the ordering operator must be represented by an ordered set of states or an ordered set of transition matrices, analogous to a tensor. Indeed, when we consider generalized ordering operators in which the DAG is literally modifed by the interaction of multiple ordering operators, we see that this analogy is precise and allows us to reinterpret the abstract symbolic tensor calculus in discrete, finite, process terms.

In general, note that each directed acyclic graph may be associated with a large number of ordering operators, just as there are many DAGs for n labeled nodes. In particular, for n labeled nodes the number of DAGs is given by the recurrence relation:

$$a_n \;=\; (k-1)\, 2^{k(n-k)}\, a_{n-k}\,.$$

The same numbers count the (0,1) matrices in which all eigenvalues are positive real numbers[5]. The proof is bijective: a matrix A is an adjacency matrix of a DAG if and only if the eigenvalues of the (0,1) matrix $A+I$ are positive, where I denotes the identity matrix.

Representing an ordering operator as a sequence of transition matrices enables us to apply a specific ordering operator to a class of DAGs in addition to the

[5]See McKay, B.D., *et al* (2004).

CDAG. For example, multiple subnets may be isomorphic to the ordering operator's canonical DAG or a subnet of it. Alternatively, a DAG may be isomorphic to the canonical DAG up to some transformation T. That is, if the adjacency matrix that results when all subnets having a particular adjacency matrix are replaced by a new subnet is isomorphic to the adjacency matrix of the canonical DAG, then the DAG may be said to be isomorphic to the canonical DAG up to the transformation T. A transformation of particular interest is one that replaces the given subnet with a single node having the inbound and outbound arcs of the subnet. Such a transformation will be called a *reduction* and the given subnet the *reduction subgraph*[6].

A specific walk of G can be understood as an ordered sequence of pairs of state vectors and adjacency sub-matrices:

$$S_c^{n+1} \;=\; \sum S_c^n \times A_{r,c}^n \,.$$

B. Properties and Objects

Following Leibnitz (identity of indiscernables), we treat properties as fundamental and objects are being a confluence of properties rather than having *a priori* existence. OOC represents a discrete, finite, process system as an ordering operator O with a specific CDAG and each occurrence of a specific property P in that system as an occurrence of a subgraph in the CDAG. Thus, every ordering operator generates a set of one or more properties and, for sufficiently complex CDAG, there may be many occurrences of a property.

The occurrences of each property P in a CDAG may be partially ordered and in a variety of ways, only one of which corresponds to the partial ordering of generation of P by the ordering operator O. Every other partial ordering corresponds to a walk of the graph, but may violate the acyclicity provided by O, effectively inducing cycles.

The co-occurrence of collection of properties having the same partial ordering for some subset of the occurrences of each property in a graph define multiple occurrences of an object defined by those properties. In set theoretic terms, we would say that the collection of properties are the defining or required properties of the set (a.k.a. the meaning criteria). A predicate corresponding to the requirement of these properties is then the membership function of the set. An instance of an object may be inferred ("exists" in an ontological sense) only by virtue of the co-occurrence of the defining properties for that object.

By suitable graph reduction, the occurrences of an object may be seen to be ordered in various ways. If they are totally ordered, we say the ordering is "time-like" and if partially ordered, then "space-like". When we say that an object is

[6]The problem of finding all similar subgraphs among two given graphs is computationally NP-complete, but we will not have need of that computation here. The problem is approached differently: we generate a hierarchy of graphs, each of which is a reduction of the previous, wherein the reduction subgraph is specified in advance.

moving through spacetime, we mean that its invariant, defining properties can be found in multiple subgraphs and that each instance is associated with properties that satisfy some definition of spacetime (such as a metric) across some ordering of those instances.

C. Metrics and Probability

Measures and metrics are inherent in the combinatorial constructions of OOC. In the generation of a CDAG, the occurrences of a particular subgraph may be counted. Since the graph is finite, the total number of occurrences is known in advance. Each ordered occurrence provides a ratio which can be interpreted as a relative frequency or, equivalently, as a distance measure on the graph. Thus, OOC unifies probability and metrics, and provides a multi-connected topology with many metrics, each specific to the property (subgraph) or properties being measured. This unification has many useful applications in quantum theory (*e.g.*, understanding EPR), but requires finite constructions.

D. Composition via Tensor Product

For graphs of sufficient complexity, there are many possible decompositions. The resulting decomposition graphs need not be acyclic. When the decomposition graphs are independent, we call them *projections*. Consider a CDAG corresponding to an ordering operator O defined as the tensor product of n independent graphs $P_1 \otimes P_2 \otimes \cdots \otimes P_n$, each with corresponding ordering operator. The tensor product then corresponds to a tensor operator in n-dimensional space and, simultaneously, to an ordering operator.

V. Correspondences to Physics

A. General Considerations

The principle of property confluence (or "co-occurrence") outlined above is similar to how we would recognize macroscopic objects. It is even how we recognize particles between two particle events in particle physics: if the track matches a feasible trajectory and the events obey the appropriate conservation laws, we assume particle identification and continuity. As an ontological principle, objects have existence only in terms of a confluence of defining properties. In physics, those properties may be quantum numbers and other invariants as represented by conservation laws. This is a kind of Wheeler-Feynman rule combined with the Eddington idea that particles are conceptual carriers between events of properties (quantum numbers) that satisfy invariants (conservation laws). A key insight is that causal structure – and therefore our notions of time and space – are defined in this way *via* Lorentz invariance.

The generation order of the directed acyclic graph generated by an ordering operator is not to be identified with time. Neither is the distance (by whatever metric) between two subgraphs A and B to be identified with spatial separation *per se*.

Instead, we find instances of the confluence of defining properties as subgraphs and establish an ordering on these subgraphs that satisfies spatio-temporal properties such as Lorentz invariant metric. The requirements for such metrics have been shown to be intrinsic to OOC[7]. In a sufficiently complex graph there will be many spatio-termporal "paths" between object instances.

B. Feynman's Discrete Path Integrals

If the spatio-temporal paths are given a representation in action-time, the graph ceases to be acyclic so that some paths are generated "backwards in time". Furthermore, it is clear that there will be a path that corresponds to a minimum of the summed action along the path – that is, a classical trajectory. We can characterize alternative paths as being separated by a "phase" that characterizes the difference in action. This representation is coordinate free and contravariant – hence the corresponding ordering operator representation is an ordered set of adjacency matrices.

Feynman and Hibbs showed how to construct the one-dimensional discrete sum over all paths for the two slit. They treated this construction as being in one spatial dimension with steps occurring in time, and no one has been able to extend it to three spatial dimensions so that the correct results are obtained. If instead of taking the Feynmann and Hibbs construction as being one-dimensional and needing to be extended, we can take the construction to be on a spatial dimension along the classical trajectory (*i.e.*, in the preferred coordinate frame of the "particle"). A different strategy now becomes apparent – we want to decompose Feynman's construction into three independent generators, *i.e.*, the projections of the paths into three coordinate bases. To put it another way, the ordering operator generates the phase space representation in "action-generation" space. The coordinate spacetime represenation must be derived from this *via* graph decomposition and reduction.

By definition, such decomposition must be a possible physical representation. Feynmans paths in phase space become real: it is the imposition of classical spacetime that is artifactual. In essence, Feynman solved the three dimensional discrete sum over all paths problem without realizing it. Furthermore, when viewed through the classical spacetime lens, the generative order on the graph appears to have an intrinsic Zitterbewegung. As explained in FDP, this Zitterbewegung satisfies the requirements for a metric with Lorentz invariance. The construction is discrete, has the required quantum mechanical properties, and is born relativistic. Even more important, it is born without infinities.

[7]See, for example, FDP.

This notion can be illustrated by considering a hypothetical ordering operator that generates a DAG in which nodes represent an interaction event among elementary particles. To be a pair of events associated with a classical particle trajectory, the pair of events must conserve certain physical properties such as energy and momentum and must satisfy certain relations such as the Lorentz transformation. These conditions allow us to "reverse engineer" spacetime on the DAG. However, it should be clear that neither space nor time are then directly generated by the ordering operator. Instead, it is imposed in hindsight as a way of organizing the information carried by the DAG. In some ways, it is closer to a discrete version of Feynman's sum over all paths and, in fact, any recursive generator of Feynman's discrete sum over all paths in 1+1 dimensions (see Feynman's and Hibbs' derivation of the solution to the 1+1 Dirac equation) can be seen as a special case of an ordering operator. The ordering operator calculus allows us to derive the propagator (see FDP), and to generalize the discrete sum over all paths into four dimensional spacetime as outlined here.

Note that the ordering operator representation provides a deterministic history, but a probabalistic future in which there are a finite (though possibly very large) number of multiple possible trajectories. As each observable event is generated, these multiple potential trajectories are reduced to a single trajectory which satisfies the relevant conservation laws (constraints) in such a way that is consistent with the classical trajectory constructed so far. In physics, we infer that a particle "carries" the conserved quantities between events. An inexact and incomplete way of characterizing this representation in causal terms might be to consider observation to "collapse" the state of this inferred particle.

A similar process occurs in the generation and recognition of linguistic events. Within a given corpus, the recipient (hearer or reader) of linguistic signals can from time to time predict with certainty the next signal, whether that signal is a phoneme, a word, a phrase (noun, verb, adverbial, adjectival), or a sentence. In between these fully determined events, there are multiple possibilities. Note that a hierarchy of overlapping "state waves" are being co-generated.

C. Interpretation of Tensor Calculus

A tensor may have either covariant or contravariant components. These correspond to the ordering operator representations as a sequence of state vectors *vs.* a sequence of adjacency matrices. As with tensors, a composition may be mixed. As representations of physical systems, the tensor calculus deals with basis transformations (including coordinate bases). These are special cases of ordering operators (as prescribed in FDP), most often derivable as projections and reductions of an ordering operator having a graph in an abstract action-generation space.

If we understand the notation of the tensor calculus in terms of state vectors and adjacency matrices having an n-dimensional basis and a metric based on occurrences of properties (subgraphs), it is obvious that the tensor calculus can represent the much more general ordering operators with similar rules of interpretation. Although the underlying state vectors and adjacency matrices will be binary, property metrics introduce non-binary measures and both state vectors and adjacency matrices in the resulting representation become the more familiar tensors. Characteristic numbers of the underlying combinatorial structures become connection coefficients. Inasmuch as it has been previously shown (FDP) that the OOC is a covering theory for differential geometry, where differential geometry is understood as approximating the high cardinality, finite combinatorics of OOC, we can reasonably anticipate a precise correspondence between ordering operators and tensors.

The tensor calculus may be understood as a sub-theory of OOC, with OOC being capable of capturing relationships outside conventional basis representations. While the tensor calculus encourages representations in a topology which is singly-connected, OOC provides representations in a multi-connected topology. As such, relationships outside the Lorentzian causal structure become not only possible, but natural.

VI. Conclusion

According to James Gleick, John Archibald Wheeler left behind "an agenda for quantum information science". We repeat this agenda here, annotated with comments relating the steps of the agenda to progress in OOC:

1. Go beyond Wootters and determine what, if anything, has to be added to distinguishability and complementarity to obtain all of standard quantum theory.

Comment: OOC provides a combinatorial theory of distinguishability and explains complementarity as a natural property of such finite and discrete process systems.

2. Translate the quantum versions of string theory and of Einstein's geometrodynamics from the language of continuum to the language of bit.

Comment: Although we eschew much of string theory as an unnecessary and obfuscating complication, previous work has addressed much of relativistic quantum theory, combinatorially deriving the fine structure constant, propagator, uncertainty, and the relativtistic Schrödinger for the hydrogen atom, and the present paper makes connection with geometrodynamics.

3. Sharpen the concept of bit. Determine whether "an elementary quantum phenomenon brought to a close by an irreversible act of amplication" has at bottom (1) the 0-or-1 sharpness of definition of bit number in a string of binary digits, or (2) the accordion property of a mathematical theorem, the length of which, that is, the number of supplementary lemmas contained in which, the analyst can stretch or shrink according to his convenience.

Comment: TBD. However, it is clear that physics is more a theory about information and its representation of knowledge, and less about some ontological reality.

While I find no reason to accept the constraints of the Copenhagen interpretation, it does seem likely that information is more context dependent than not, and so a bit more like (2) in this regard.

4. Survey one by one with an imaginative eye the powerful tools that mathematics – including mathematical logic – has won and now offers to deal with theorems on a wholesale rather than a retail level, and for each such technique work out the transcription into the world of bits. Give special attention to one and another self-referential deductive system.

Comment: Many of those mathematical tools rely upon features that are inconsistent with finitism, discreteness, and process-orientation and so, from my perspective,are incompatible with "the world of bits".

5. From the wheels-upon-wheels-upon-wheels evolution of computer programming dig out, systematize and display every feature that illuminates the level-upon-level-upon level structure of physics.

Comment: This is precisely the importance of the Combinatorial Hierarchy, and of OOC decomposition and reduction. There are other combinatorial and hierarchical relationships of importance as well, too numerous to go into here.

6. Capitalize on the findings and outlooks of information theory, algorithmic entropy, evolution of organizisms, and pattern recognition. Search out every link that each has with physics at the quantum level. Consider, for instance, the string of bits 1111111... and its representation as the sum of the two strings 1001110... and 0110001... Explore and exploit the connection between this information-theoretic statement and the finding of theory and experiment on the correlation between the polarizations of the two photons emitted in the annihilation of singlet positronium and in like Einstein-Podolsky-Rosen experiements. Seek out, moreover, every realization in the realm of physics of the information-theoretic triangle inequality recently discovered by Zurek.

Comment: These relationships have been explored in previous papers and OOC has been shown to accommodate if not explain (previously characterized as "simulate") EPR results. Prof. Noyes' Bit String Physics provides a bit string representation of the standard model of particle physics and a bit string can be understood as a projection of an ordering operator with respect to quantum numbers. I believe that the greater richness of graphs is necessary for a complete model, especially if quantum general relativity is to be incorporated. Much remains to be done.

7. Finally. Deplore? No, celebrate the absence of a clean clear definition of the term 'bit' as elementary unit in the establishment of meaning. We reject "that view of science which used to say, 'Define your terms before you proceed'. The truly creative nature of any forward step in human knowledge," we know, "is such that theory, concept, law, and method of measurementforever inseparableare born into the world in union." If and when we learn how to combine bits in fantastically large numbers to obtain what we call existence, we will know better what we mean both

by bit and by existence.

Comment: Here we disagree with Wheeler in part and insist that our terms be defined before we proceed. On the other hand, Wheeler may well have been concerned that such a stricture was too rigid to accommodate learning and refinement. Our solution (see FDP) is to engage in a methodology that permits definitions of terms and their relationships to be refined iteratively, mimicking the idealized scientific method. Interestingly, this process can be modeled using OOC and so is consistent with it. Our goal is like Wheeler's – in OOC, "theory, concept, law, and method of measurementforever inseparableare born into the world in union."

All the foregoing point to much work to be done, putting flesh on the skelaton as it were and refining the interpretation of tensor calculus notation. However, the results obtained to date and the ability of OOC to represent both intrinsically discrete theories such as quantum mechanics and intrinsically continuum theories such as geometrodynamics is encouraging.

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Brief Biography

David McGoveran is founder and principal of Alternative Technologies, a consulting firm specializing in emerging software technologies and theoretical contributions to computer science. Educated in the foundations of physics, mathematics, and logic, he has periodically held visitor positions with the Theory Group, SLAC under the sponsorship of Prof. Noyes (since the mid-1980s) and has joyfully collaborated whenever possible with Prof. Noyes over the last thirty years.